

3.1 Linear Equations and Arithmetic Sequences

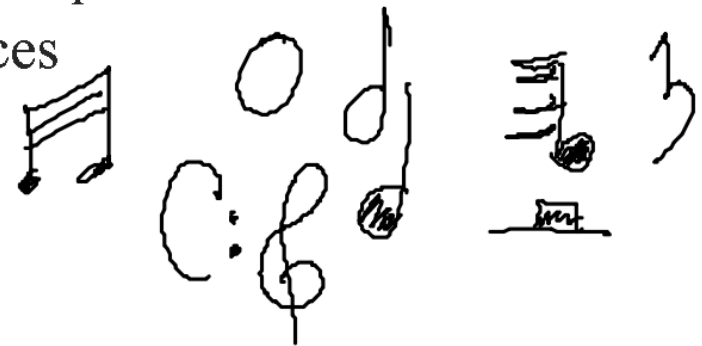
$$u_0 = 2.27$$

$$u_n = u_{n-1} + 1.37$$

$$u_n = u_0 + (n-1)d$$

$$u_n = 2.27 + (30-1)1.37$$

$$u_n = \$412$$



$$y = 2.27 + 1.37x$$

linear equation

explicit formula

$$u_n = 2.27 + 1.37n$$

Ex A

$$u_0 = 2$$

$$u_n = u_{n-1} + 6$$

where $n \geq 1$

$$u_n = 2 + 6n \quad \text{explicit formula}$$

$$u_{22} = 2 + 6(22)$$

$$u_{22} = 134$$

$$u_n = 86 \quad n = ?$$

$$\underset{-2}{86} = \underset{-2}{2} + 6n$$

$$\frac{84}{6} = \frac{6n}{6} \quad n = 14$$

$$d = \Delta y$$

$$1 = \Delta x$$

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

$$\frac{6}{1} = 6$$

$$d = \text{slope}$$

$$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(1, 8) \quad (3, 20)$$

$$(x, y)$$

$$\frac{20 - 8}{3 - 1} = \frac{12}{2} = 6$$

$$u_0 = 2$$

$$u_n = u_{n-1} + 6$$

y-intercept $x=0$
where it hits the
y-axis

$$(0, 2)$$

$$(n, u_n)$$

$$y = 2 + 6x$$

$$y = a + bx$$

\nearrow \nearrow
 y-intercept slope

Slope-intercept form

10/17

Inv.

Matching

- | | | |
|---|---|---------------|
| 1 | B | $y = 4 - x $ |
| 2 | C | iii |
| 3 | | i |

Ex B $U_1 = 17$
 $U_n = U_{n-1} - 2$
where $n > 1$

Typically spends \$2
per day on lunch.

a) $U_n = 17 - 2n$
 $y = 17 - 2x$

b)

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#1-6

$$\text{slope} = \frac{\Delta y}{\Delta x} \quad \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept form

$$y = a + bx$$

↑ ↑
y-intercept slope

independent variable - (x) the variable you change or control

domain - all the x's for the problem

dependent variable - (y) the variable you measure

range - all the y's

Ex $20 \frac{\text{mi}}{\text{gal}}$

full = 16.4 gal

a) Ind mi
Dep gal

b) $y = 16.4 + 20x$

c) $y = 16.4 + 20(175)$ $y = 3516.4 \text{ gal}$

$$y(\text{gal}) = 16.4(\text{gal}) - 20 \frac{\text{mi}}{\text{gal}} (175 \text{ mi})$$

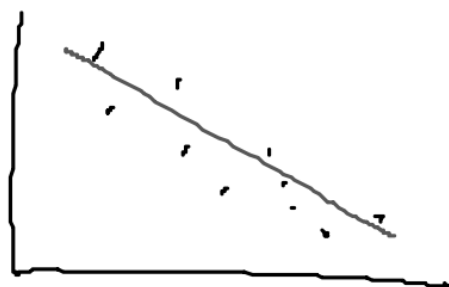
$$\frac{1}{20} \frac{\text{gal}}{\text{mi}} (175 \text{ mi})$$

$$x = 16.4 - 0.05(175) = 7.65 \text{ gal}$$

d) $2 = 16.4 - 0.05x = 288 \text{ mi}$

P. 124 #1-5

Line of Fit



- Line must follow data pattern
- Should divide data points approximately equally

Point-slope Form

(x_1, y_1) (x_2, y_2)

$$b = \frac{y_2 - y_1}{x_2 - x_1} \text{ slope}$$

$$(y - y_1) = b(x - x_1)$$

$$y = b(x - x_1) + y_1$$

$(2, 3)$ $(4, 5)$

$$b = \frac{5 - 3}{4 - 2} = \frac{2}{2} = 1$$

$$* y - 5 = 1(x - 4)$$

$$* y = (x - 4) + 5$$

$$y = x - 4 + 5$$

$$* y = x + 1$$

Ex Find a line of fit

$$(1994, 359.0) \quad (\underline{1984}, \underline{343.5})$$

$$b = \frac{359.0 - 343.5}{1994 - 1984} = \frac{15.5}{10} = \underline{1.55}$$

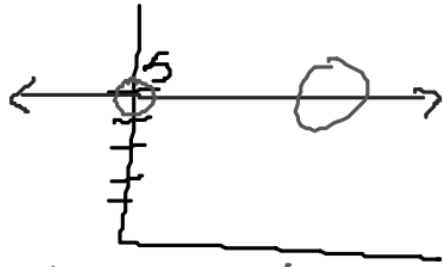
$$a) \quad y = \underline{343.5} + \underline{1.55}(x - \underline{1984})$$

$$y = 343.5 + 1.55(2050 - 1984)$$

$$b) \quad y = 445.8 \text{ ppm CO}_2$$

Extrapolation - finding data that falls
outside of your known points

interpolation - " "
inside " "



$(0, 5) (, 5)$

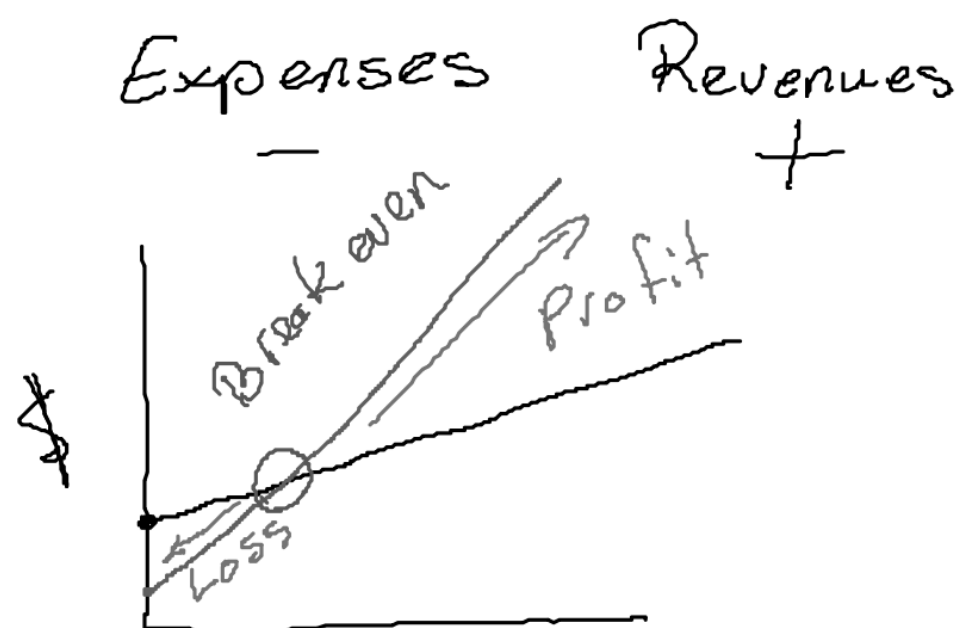
$$\frac{5-5}{5-0} = \frac{0}{5} = 0$$

Parallel \Rightarrow same slope

$$y = y_1 + b(x - x_1)$$

2a) slope $\frac{2}{3}$ $(5, -7)$

$$y = -7 + \frac{2}{3}(x - 5)$$



Ex A

PPP

1st min 20¢
ea 17¢

x min after first
y total cost

$$y = 20 + 17x$$

$$\begin{array}{r} y = 20 + 17x \\ - (y = 50 + 11x) \\ \hline \end{array}$$

$$\begin{array}{r} 0 = -30 + 6x \\ +30 \quad +30 \end{array}$$

$$\begin{array}{r} 30 = 6x \\ \underline{6} \quad \underline{6} \end{array}$$

$$x = 5 \text{ min}$$

SBP

50¢
11¢

$$y = 50 + 11(x-1)$$

$$y = 20 + 17(5)$$

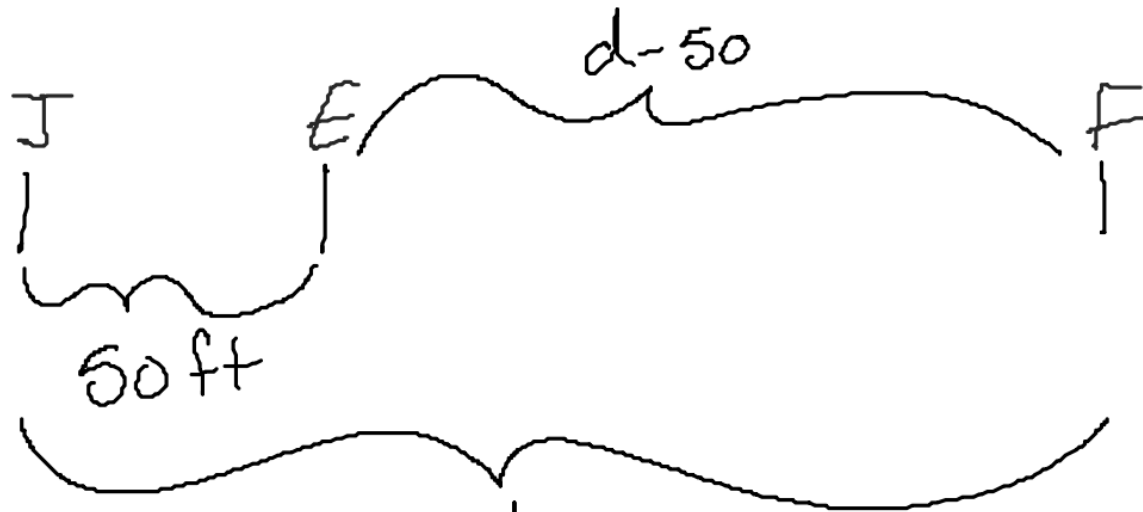
$$y = 105¢$$

$$x < 5 \quad \text{PPP}$$

$$x > 5 \quad \text{SBP}$$

Ex B

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#1, 3, 4, 5



$$E \ 12.5 \text{ ft/s}$$

$$J \ 14.3 \text{ ft/s}$$

$$t = \text{time (s)}$$

$$J \ d = 14.3t$$

$$E \ d - 50 = 12.5t$$

$$d = 12.5t + 50$$

$$d = 14.3(27.8) \quad 14.3t = 12.5t + 50$$

$$d = 397.54 \text{ ft} \quad \begin{array}{r} -12.5t \quad -12.5t \\ \hline 1.8t = 50 \end{array}$$

$$t = 27.8 \text{ s}$$

$$3) \perp y = 4 - 2.5x \quad (1, 5)$$

$$\text{slope} = -2.5$$

→ \perp change sign & flip the fraction

$$-\frac{5}{2} \perp +\frac{2}{5}$$

$$y = y_1 + \frac{2}{5}(x - x_1)$$

$$y = 5 + \frac{2}{5}(x - 1)$$

$$\begin{aligned} \text{1a)} \quad y &= 3x - 17 \\ y &= -2x - 8 \end{aligned}$$

$$3x - 17 = -2x - 8$$

3.7 Substitution and Elimination

Ex. A $y = 15 + 8x$
 $-10x - 5y = -30$

$$-10x - 5(15 + 8x) = -30$$

$$-10x - 75 - 40x = -30$$

$$\begin{array}{r} -75 - 50x = -30 \\ +75 \qquad \qquad +75 \end{array}$$

$$\frac{-50x}{-50} = \frac{45}{-50}$$

$$x = -\frac{9}{10}$$

$$y = 15 + 8\left(-\frac{9}{10}\right)$$

$$y = 15 - \frac{72}{10}$$

$$y = 7.8$$

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$$\begin{array}{r} 5(y = 15 + 8x) \\ -5y = -30 + 10x \end{array}$$

$$+ \frac{5y = 75 + 40x}{\hline}$$

$$\begin{array}{r} 0 = 45 + 50x \\ -45 \quad -45 \end{array}$$

$$\frac{-45}{50} = \frac{50x}{50}$$

$$-\frac{9}{10} = x$$

elimination

$$\text{Ex B } 4x + 3y = 14$$

$$3x - 3y = 13$$

$$7x \quad 0y \quad 27$$

$$\frac{7x}{7} = \frac{27}{7}$$

$$x = \frac{27}{7}$$

$$\left(\frac{27}{7}, -\frac{10}{21} \right)$$

$$y = -\frac{10}{21}$$

Elimination:)

$$4\left(\frac{27}{7}\right) + 3y = 14$$

$$\frac{3y}{3} = \left(14 - 4\left(\frac{27}{7}\right) \right) \frac{1}{3}$$

$$y = \frac{14}{3} - \frac{36}{7}$$

$$y = \frac{98 - 108}{21}$$

$$-3x + 5y = 6$$

$$2x + y = 6$$

$$y = 6 - 2x$$

$$-3x + 5(6 - 2x) = 6$$

$$-3x + 30 - 10x = 6$$

$$-13x + 30 = 6$$

$$\quad -30 \quad -30$$

$$\begin{array}{r} -13x = -24 \\ \hline -13 \quad -13 \\ \hline x = \frac{-24}{-13} \\ x = \frac{24}{13} \end{array}$$

$$y = 6 - 2\left(\frac{24}{13}\right)$$

$$y = 6 - \frac{48}{13}$$

$$y = \frac{78}{13} - \frac{48}{13} = \frac{30}{13}$$

$$\left(\frac{24}{13}, \frac{30}{13}\right)$$

Yay

$$\text{Inv } 7x + 2y = -3 \quad 2y = -3 - 7x$$

$$3x + 4y = 5 \quad y = -\frac{3}{2} - \frac{7}{2}x$$

$$4y = 5 - 3x$$

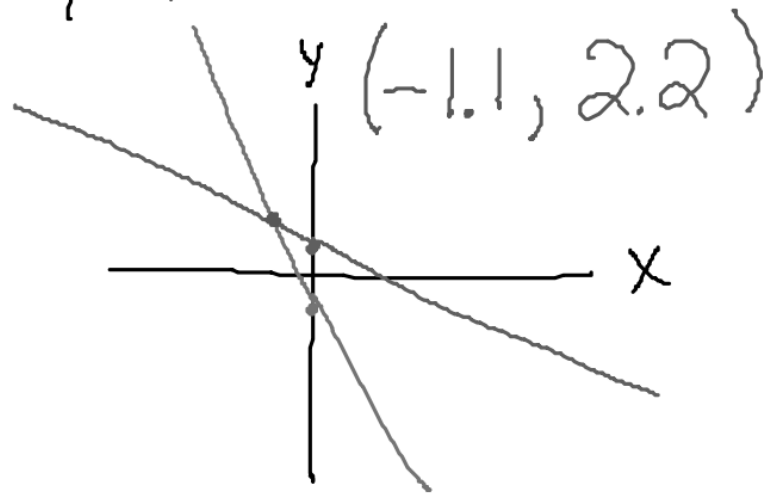
$$y_2 = (5 - 3x)/4 \quad y_1 = (-3 - 7x)/2$$

$$x_{\min} -10$$

$$x_{\max} 10$$

$$y_{\min} -10$$

$$y_{\max} 10$$



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#1, 2, 3, 5, 6

$$1b) \quad \begin{array}{rcl} 2p + 3h & = & 18 \\ -2p & & -2p \end{array}$$

$$\frac{3h}{3} = \frac{18}{3} - \frac{2p}{3}$$

$$h = 6 - \frac{2}{3}p$$

$$5b) \quad \frac{1}{4}x - \frac{2}{5}y = 3$$

$$\frac{3}{8}x + \frac{2}{5}y = 2$$

$$\frac{8}{5} \frac{5}{8}x = 5 \frac{8}{5}$$

$$x = 8 \quad \left(8, -\frac{5}{2}\right)$$

$$\frac{1}{4}(8) - \frac{2}{5}y = 3$$

$$-2 - \frac{5}{2} - \frac{2}{5}y = 3 - \frac{5}{2}$$

$$y = -\frac{5}{2}$$

$$j + 5k = 8$$

$$\begin{array}{rcl} j & = & 8 - 5k \\ -3j & - & 15k = -24 \end{array}$$

$$\frac{-3j}{-3} = \frac{-24}{-3} + \frac{15k}{-3}$$

$$j = 8 - 5k$$

The same line.

$$8x + 7y = 14$$

$$4(2x - 3y = 7)$$

$$-(8x - 12y = 28)$$

$$\frac{0x + 19y}{19y} = -14$$

$$5c) \begin{cases} 4x + 9y = 12 & \times 3 \\ 3x - 8y = 10 & \times 4 \end{cases}$$

$$\begin{array}{r} 12x + 27y = 36 \\ - (12x - 32y = 40) \\ \hline \end{array}$$

$$\frac{59y}{59} = \frac{-4}{59}$$

$$y = -\frac{4}{59}$$

$$4x + 9\left(-\frac{4}{59}\right) = 12$$

$$4x + -\frac{36}{59} = 12$$

$$\frac{4x}{4} = \frac{12 + \frac{36}{59}}{4}$$

$$\begin{cases} 4x + 9y = 12 & \times 3 \\ 3x - 8y = 10 & \times -4 \end{cases}$$

$$\begin{array}{r} 12x + 27y = 36 \\ -12x + 32y = 40 \\ \hline \end{array}$$

$$0x + 59y = -4$$

$$x = 3 + \frac{9}{59} \quad \frac{177}{59}$$

$$x = \frac{186}{59} \text{ or } 3\frac{9}{59}$$

$$\left(\frac{186}{59} \mid \frac{-4}{59} \right)$$

$$\begin{aligned}
 5e) \quad f &= 3d + 5 \\
 10d - 4f &= 16 \\
 10d - 4(3d + 5) &= 16 \\
 10d - 12d - 20 &= 16 \\
 -2d - 20 &= 16 \\
 \quad +20 \quad +20 & \\
 \hline
 -2d &= 36 \\
 \quad -2 \quad -2 &
 \end{aligned}$$

$$d = -18$$

$$f = 3(-18) + 5$$

$$= -54 + 5$$

$$f = -49$$

$$(-18, -49)$$

$$\begin{aligned}
 3 \left(\frac{1}{4}x - \frac{4}{5}y = 7 \right) \\
 \frac{3}{4}x + \frac{2}{5}y = 2 \\
 - \left(\frac{3}{4}x - \frac{12}{5}y = 21 \right) \\
 \hline
 0x + \frac{14}{5}y = -19 \\
 y = \frac{-19 \cdot 5}{1 \cdot 14} = \frac{-95}{14}
 \end{aligned}$$

$$\frac{1}{4}x - \frac{4}{5}\left(-\frac{95}{14}\right) = 7$$

$$4x + y = 6$$

$$(4x + y - 3)^2 = ?$$

$$(6 - 3)^2 = 3^2 = 9$$

$$4x + 3y = 14$$

$$3x - 3y = 13$$

$$7x + 0y = 27$$

$$7x = ?$$

1-8, 11, 12, 17, 19, 20

$$7a) \begin{array}{l} (5x - 4y = 5) \cdot 2 \\ (2x + 10y = 2) \cdot 5 \end{array}$$

$$\begin{array}{r} 10x - 8y = 10 \\ - (10x + 50y = 10) \\ \hline \end{array}$$

$$\text{Elimination} \quad \begin{array}{r} 0x - 58y = 0 \\ \quad \quad \quad -58 \quad \quad -58 \end{array}$$

$$y = 0$$

$$2x + 10(0) = 2 \quad (1, 0)$$

$$\frac{2x}{2} = \frac{2}{2}$$

$$x = 1$$

$$6b) \quad y = \frac{3}{4}x - 1$$

$$\frac{7}{10}x + \frac{2}{5}y = 8$$

$$\frac{7}{10}x + \frac{2}{5}\left(\frac{3}{4}x - 1\right) = 8$$

$$\frac{7}{10}x + \frac{6}{20}x - \frac{2}{5} = 8$$

$$\frac{10}{10}x - \frac{2}{5} = 8$$

$$x = 8\frac{2}{5}$$